Turbulent convection over a heated horizontal surface

By D. B. THOMAS and A. A. TOWNSEND
Cavendish Laboratory, Cambridge

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SUMMARY

Detailed measurements have been made of the temperature field in natural convection above a heated horizontal surface in air, with and without a cold upper boundary. The object was to provide experimental material for the testing of existing theories of convection and the development of new ones. Using a variety of experimental techniques, it was possible to measure the heat transfer, the mean temperature profiles, mean squares of the temperature fluctuations and the autocorrelation functions of the temperature fluctuations. These results are considered in relation with the 'similarity' theory of Priestley and the 'neutral stability' theory of Malkus. Both these theories lead to the conclusion that, for convection between parallel planes at high Rayleigh numbers, nearly the whole of the mean temperature variation occurs in comparatively thin surface layers whose mutual interaction is small. The present experiments confirm this in some detail, but the exact form of the variation of mean temperature cannot be completely reconciled with either of the theories.

1. INTRODUCTION

The motion set up in a fluid which is enclosed by two plane horizontal boundaries and heated from below forms a very suitable subject for both theoretical and experimental study of heat convection, the boundary conditions being homogeneous in the horizontal directions. As the difference of temperature between the surfaces increases from zero, an ordered, cellular motion usually appears at a Rayleigh number about the predicted value of 1708, but this regular motion disappears and is replaced by a random 'turbulent' motion at some Rayleigh number around 50,000 (Jakob 1949; de Graaf & van der Held 1953). More recent work by Malkus (1954 a) has shown that there may be as many as five transitions between one flow regime and another before fully turbulent convection is established for Rayleigh numbers exceeding one million.

Apart from some observations of the motion of suspended particles (Malkus 1954 a), there have been few attempts to study any other aspects of the problem than the dependence of heat transfer coefficient on Rayleigh number, and the experimental work to be described was undertaken in the hope of providing more detailed information about conditions within the fluid. Working with air, it has been possible to measure mean temperatures and temperature fluctuations at points between the surfaces and so to make a more stringent test of the several theoretical predictions than would be possible with heat transfer measurements alone. Similar measurements have been made in the convection field above a single heated plate. This last arrangement is an approximation to the convection over an infinite horizontal surface and may be compared with the predictions of the similarity theory (Priestley 1954). It may also resemble the convection near the lower of two parallel planes at very high Rayleigh numbers.

2. The equations of motion and of heat

Let us consider the convection in a perfect gas in the space above a rigid horizontal surface whose temperature is everywhere T_1, measured in the absolute gas scale. The coordinate system is chosen so that the origin is in the plane of the surface and Oz is vertically upwards. Sometimes the fluid is bounded above by a similar horizontal surface in the plane $z = D$, and with temperature T_2. If there is no such upper boundary, the temperature at a great distance from the surface is T_a. Then

u, v, w	are the velocity components parallel to Ox, Oy, Oz, $q^2 = u^2+v^2+w^2$,
P, p	are the mean pressure and the fluctuation about the mean,
T, θ	are the mean temperature and the fluctuation about the mean,
γ	is the ratio of the specific heats,
ν	is the kinematic viscosity,
κ	is the thermometric conductivity,
H	is the constant upward flux of total heat,
$Q = H/(\rho c_p T) = (\gamma-1)H/\gamma P$	is a 'kinematic' measure of the heat flux,
$A = gD^3 \log(T_1/T_2)/\nu\kappa$	is the Rayleigh number (this definition is consistent with use of the logarithm of the absolute temperature and, for numerical purposes, is nearly identical with the ordinary definition).

For motions on the laboratory scale, and with fluid velocities small compared with the velocity of sound, the constant flux of total heat is given by

$$H = -\rho c_p \kappa \frac{\partial T}{\partial z} + \rho c_p \overline{\theta w}, \tag{2.1}$$

where the single bar denotes the mean value with respect to time. This may be written as

$$Q = -\frac{\kappa}{T}\frac{\partial T}{\partial z} + \frac{\overline{\theta w}}{T}, \tag{2.2}$$

where Q is also a constant.

To the customary approximation, the equation for the turbulent energy per unit mass is

$$\frac{\partial}{\partial z}\left(\frac{1}{2}\overline{q^2 w} + \frac{1}{\rho}\overline{pw}\right) = g\frac{\overline{\theta w}}{T} - \epsilon \tag{2.3}$$

where ϵ is the rate of dissipation of energy by viscous stresses. The terms on the left represent transport of energy from one part of the flow to another, the terms on the right represent generation of energy by buoyancy forces and the viscous dissipation.

Another mean value equation of some interest is the equation for the entropy associated with the temperature fluctuations, which is $-\frac{1}{2}c_p\,\overline{\theta^2}/T^2$. To this approximation it is

$$c_p\frac{\overline{\theta w}}{T}\frac{1}{T}\frac{\partial T}{\partial z} + c_p\frac{\partial}{\partial z}\left(\frac{\overline{\theta^2 w}}{2T^2}\right) = \kappa c_p\frac{\overline{\theta\nabla^2\theta}}{T^2}, \tag{2.4}$$

showing that entropy (or temperature fluctuations) is produced at a rate proportional to the mean temperature gradient.

It may be noticed that all three mean value equations (2.2 to 2.4) involve temperature variations only as fractions of the local mean temperature, which suggests that they would be more naturally expressed in terms of the logarithm of the temperature rather than the temperature. The distinction is of no importance if the total temperature variation is negligible compared with the absolute temperature, but this is far from true in most of the experiments to be described. In the following analysis of the experimental results, the logarithm of the temperature has been used consistently with a considerable improvement in the correlation with theoretical predictions.

3. Theories of the convection

A first step in the study of any problem of fluid motion is to use dimensional analysis to obtain the forms of the functional relations between the observables. For any convection between parallel, horizontal surfaces, the boundary conditions are geometrically similar and may be specified by the separation of the surfaces and the difference of the logarithms of the temperatures. If the equations of motion, heat, and conservation of mass, are put into non-dimensional form by use of scales of length, temperature and velocity derived from D, $\log(T_1/T_2)$ and the quantities g, ν, κ, it is immediately evident that the convection depends only on the Rayleigh number and the Prandtl number, ν/κ. For example, the heat transfer coefficient is

$$\frac{QD}{\kappa\log(T_1/T_2)} = F(A, \nu/\kappa), \tag{3.1}$$

and the distribution of mean temperature is

$$\log(T/T_1) = f\left(\frac{z}{D}, A, \frac{\nu}{\kappa}\right). \tag{3.2}$$

For very large separations of the surfaces (high Rayleigh numbers), it seems possible that the convection close to each surface may be nearly independent of the presence of the other surface and so of D. The basic equations may still be made non-dimensional, and conveniently so by using the following scales of length, velocity and (logarithmic) temperature,

$$z_0 = \left(\frac{\kappa^3}{Qg}\right)^{1/4}, \qquad u_0 = (Qg\kappa)^{1/4}, \qquad \theta_0 = \left(\frac{Q^3}{\kappa g}\right)^{1/4},$$

and their solution depends only on the Prandtl number. It follows that

$$\log\frac{T_1}{T_a} = C\theta_0 = C\left(\frac{Q^3}{\kappa g}\right)^{1/4}, \tag{3.3}$$

that

$$\log\frac{T}{T_1} = \theta_0 f\left(\frac{z}{z_0}, \frac{\nu}{\kappa}\right), \tag{3.4}$$

and that

$$\frac{\overline{\theta^2}}{T^2} = \theta_0^2\, g\left(\frac{z}{z_0}, \frac{\nu}{\kappa}\right), \tag{3.5}$$

where C is a function of Prandtl number, and T_a is the asymptotic temperature far from the lower surface but not so far as to be within the range of influence of the upper surface. If this temperature exists, equation (3.3) defines the variation of heat transfer coefficient with Rayleigh number as

$$\frac{QD}{\kappa\log(T_1/T_2)} = (2C)^{-4/3}\left(\frac{\nu}{\kappa}\right)^{1/3} A^{1/3} \tag{3.6}$$

for Rayleigh numbers so high that the temperature gradient is negligible except close to the surfaces. The proportionality between heat transfer coefficient and cube root of Rayleigh number is well confirmed by experiment for Rayleigh numbers above 10^6 (e.g. Malkus 1954 a), and this provides strong support for the concept of surface convection layers within which most of the temperature variation occurs.

The concept of a surface layer with a structure determined by the heat flux through it is very similar to the concept of a constant stress layer in turbulent flow near a wall, and this suggests that there may be a part of the surface layer within which the direct effects of conductivity and viscosity are negligible. If there is such a region, within it we have

$$\log\frac{T}{T_a} = B_r\, Q^{2/3} g^{-1/3} z^{-1/3} \tag{3.7}$$

and

$$\frac{\overline{\theta^2}}{T^2} = B_0\, Q^{4/3} g^{-2/3} z^{-2/3}, \tag{3.8}$$

where the constants may depend on Prandtl number. The prediction (3.7) is supported by measurements of heat transfer in the lower atmosphere (Priestley 1954).

The only complete theory of the convection between parallel planes is that proposed by Malkus (1954 b) and suggested to him by his observations of successive transitions between the initial cellular convective flow and the final state of completely disordered turbulent motion. Malkus suggests that the flow adjusts itself to transfer the maximum amount of heat compatible with the boundary conditions and subject to the additional restrictions,

(*a*) that the heat flux is everywhere down the gradient of mean temperature,

(*b*) that the variations of temperature and vertical velocity may be represented by Fourier series which terminate after a finite number of terms. (The number of terms in the two Fourier series is determined by requiring that the distribution of mean temperature, which is defined by these conditions, should be neutrally stable with respect to perturbations of the smallest wavelength occurring in the Fourier series.)

Unlike most current theories of turbulent motion, this theory is based on generalizations more like to those of thermodynamics than those of fluid mechanics, but it has the considerable virtue of making definite numerical predictions. For our purpose the most relevant are (i) that, not too close to the bounding surfaces,

$$\frac{1}{T}\frac{dT}{dz} = -\frac{1}{2}\frac{\kappa}{QD^2}\left(\frac{\log T_1/T_2}{\sin \pi z/D}\right)^2 \tag{3.9}$$

or, more usefully,

$$\log T/T_a = -\frac{\kappa}{2\pi QD}\left(\log\frac{T_1}{T_2}\right)^2 \cot\frac{\pi z}{D} \tag{3.10}$$

where $T_a^2 = T_1 T_2$, and (ii) that the mean square of the temperature fluctuations, averaged over the whole space, is

$$\frac{\overline{\overline{\theta^2}}}{T^2} = \frac{2}{3\pi^2}\left(\log\frac{T_1}{T_2}\right)^3 \frac{\kappa}{QD}\log\frac{QD}{\kappa}, \tag{3.11}$$

where the double bar signifies a double average, first over time at a point and then over all values of z in the field. It should be remarked that the Malkus theory is consistent with the assumption of independent surface layers at high Rayleigh numbers but not with the additional assumption that the direct effects of viscosity and conductivity are negligible. By using published measurements of the heat transfer coefficient, which may be represented by equation (6.2), equation (3.10) may be written as

$$\log\frac{T}{T_a} = \frac{(2000\nu/\kappa)^{1/2}}{2\pi^2}\theta_0\left(\frac{z}{z_0}\right)^{-1}, \tag{3.12}$$

valid for values of z/z_0 within the surface layer which are not too small. This is of the same functional form as equation (3.4).

4. Experimental arrangements

The source of heat in these experiments was an electrically heated rectangular duraluminium plate of dimensions 30 cm × 40 cm × 1 cm. In order to secure a uniform distribution of surface temperature and to allow direct measurements of heat transfer, this plate formed the upper side of a duraluminium-asbestos-duraluminium sandwich (figure 1), the lower plate being heated by a grid of resistance wire wound on an asbestos former and placed against its lower side. Four copper-constantan thermocouples were inserted in each duraluminium plate, and from the mean temperature difference and the measured thermal conductivity of the asbestos thermal resistance the total heat loss from the top plate could be calculated. From this was subtracted the heat loss by radiation and by conduction along the pillars supporting the upper surface. The radiation correction was computed using the emissivity 0·24 appropriate to polished aluminium, and was found to form about 30% of the total heat loss. In the parallel plate experiments, the upper boundary was a similar aluminium plate forming part of a water jacket. The temperature of this plate could also be measured by thermocouples set in it. Both plates were ground flat after assembly and were kept at known distances apart by glass pillars ground to size. Vertical walls of rubber or asbestos were always fitted to prevent entry of air from the edges of the space. For the measurements over a single heated surface, vertical side walls, 60 cm high, were provided for the same purpose.

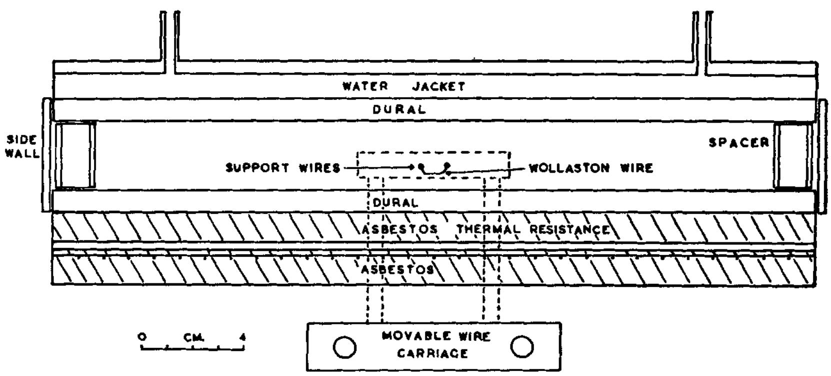

Figure 1. Experimental arrangement: diagrammatic section.

Temperature measurements in the air space were made using resistance thermometers of Wollaston wire, 0·00025 cm in diameter of measured temperature coefficient 0·00353 deg^{-1}, along which were passed measuring currents of about one milliampere. The length of the sensitive, etched portion of the wires was usually about 3 mm, giving a resistance of about 30 ohms. The response time of such a wire is of order one millisecond. The wires were soldered between two 30 s.w.g. copper wires stretched horizontally between supports on a movable carriage spanning the whole apparatus. This carriage could be moved either vertically or horizontally by micrometer screws.

The resistance of the Wollaston wires was measured with a Wheatstone bridge supplied with alternating current of 1000 c.p.s. The out-of-balance signal of the bridge was amplified and supplied to a phase-sensitive detector. The current output from such a detector is a linear function of the wire resistance and, passed through a reflecting galvanometer of period about one second, gave deflections of a light beam that could be recorded by a drum camera (see figure 2, plate 1 and figure 9, plate 2 for some sample records).

The whole system was calibrated by substituting known resistances for the thermometer element. From these records, the mean temperature was determined using a planimeter to measure the area under the trace, and the variance by measuring the deviation at intervals along the record. The duration of each record was about eight minutes.

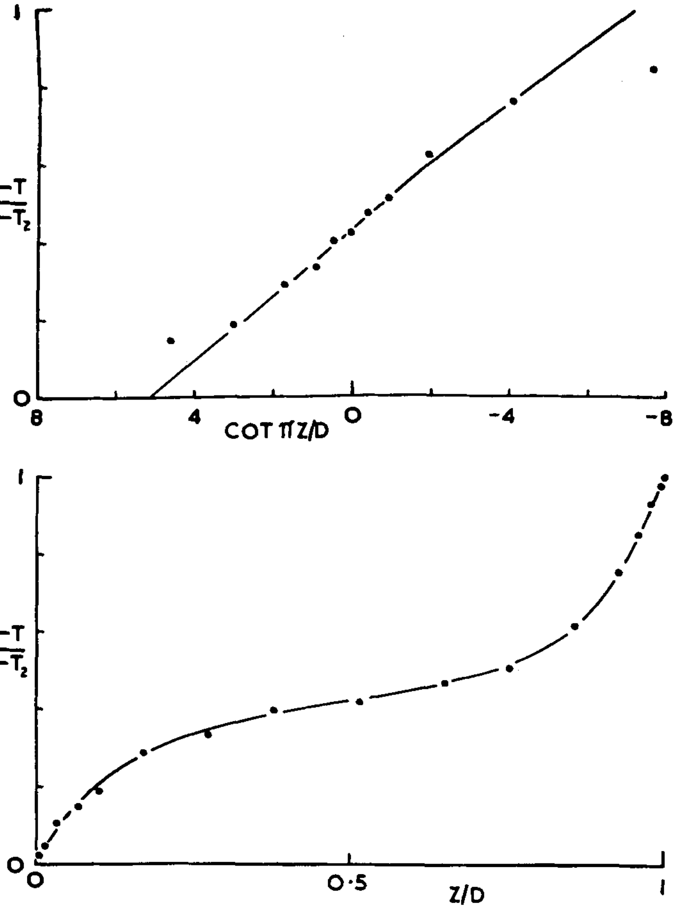

Figure 3. Distribution of mean temperature between parallel plates (D = 2·92 cm, T_1 = 52·4° C, T_2 = 12·3° C, A = 0·85 × 10^5).

In general, the supports of the wire were not at the same height as the wire itself and so were not at the same temperature. An appreciable amount of heat may be conducted along the supports and lead to a systematic error in mean temperature. A correction for this was calculated and applied to the measured values.

Some measurements of the autocorrelation function of the temperature fluctuations were also made. For this, the rectified and smoothed output from the Wheatstone bridge was used to modulate the frequency of an oscillator, and the oscillations recorded on magnetic tape. This record could be played back and the autocorrelation function determined by comparing frequencies at points on the tape separated by a known fixed distance, that is, the frequencies recorded at moments separated in time by the time necessary for the tape to travel the fixed distance. The actual comparison consists of the observation of the rate of coincidences between standard duration pulses constructed from each set of played-back oscillations, the rate being proportional to the product of the instantaneous pulse frequencies (Thomas 1956).

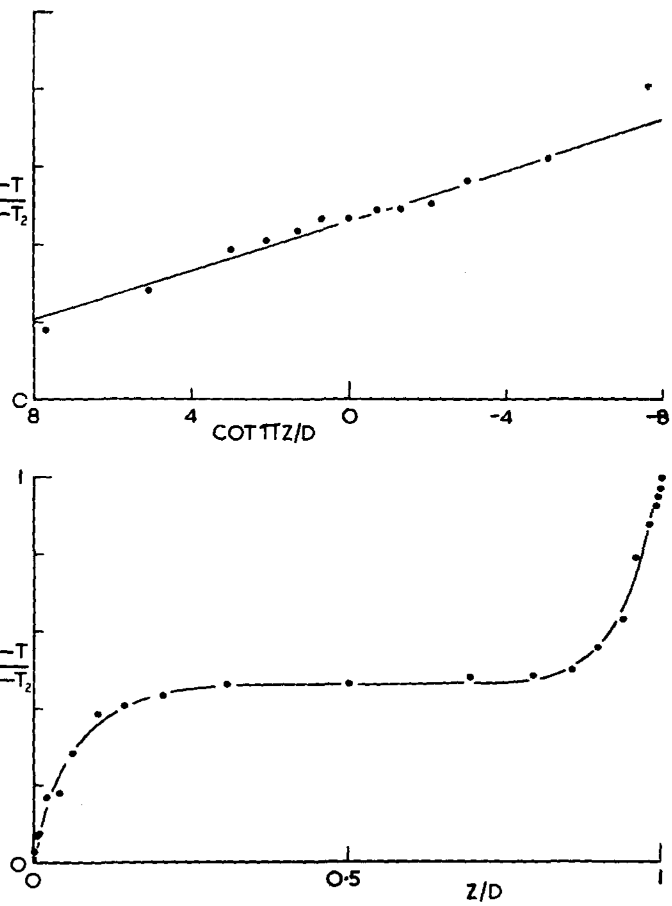

Figure 4. Distribution of mean temperature between parallel plates (D = 4·88 cm, T_1 = 46·8° C, T_2 = 10·6° C, A = 3·77 × 10^5).

5. The convection between parallel horizontal planes

Three sets of measurements were made with very nearly the same temperature difference between the plates and three different spacings. Table 1 summarizes the experimental conditions and the measured heat

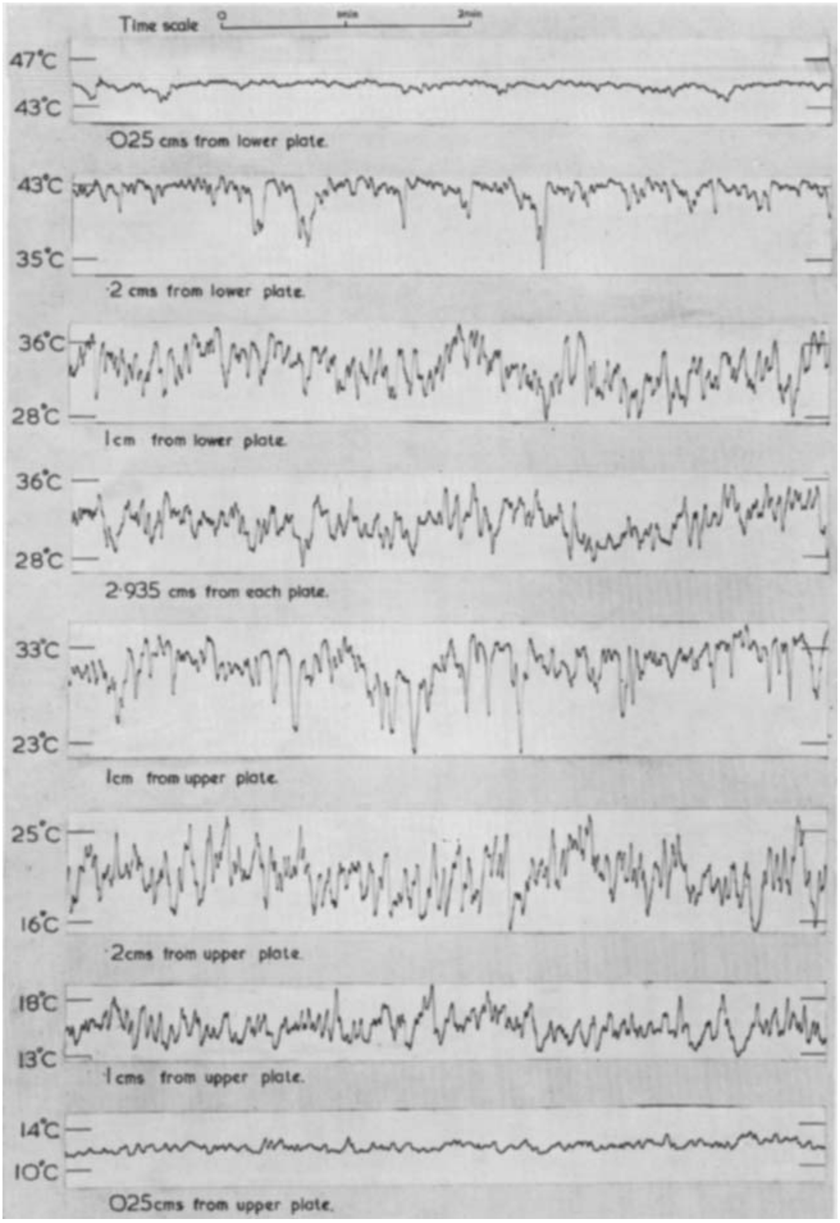

Figure 2. Temperature fluctuation records between parallel horizontal plates ($D = 5{\cdot}87$ cm, $T_1 = 46{\cdot}5°$ C, $T_2 = 9{\cdot}7°$ C).

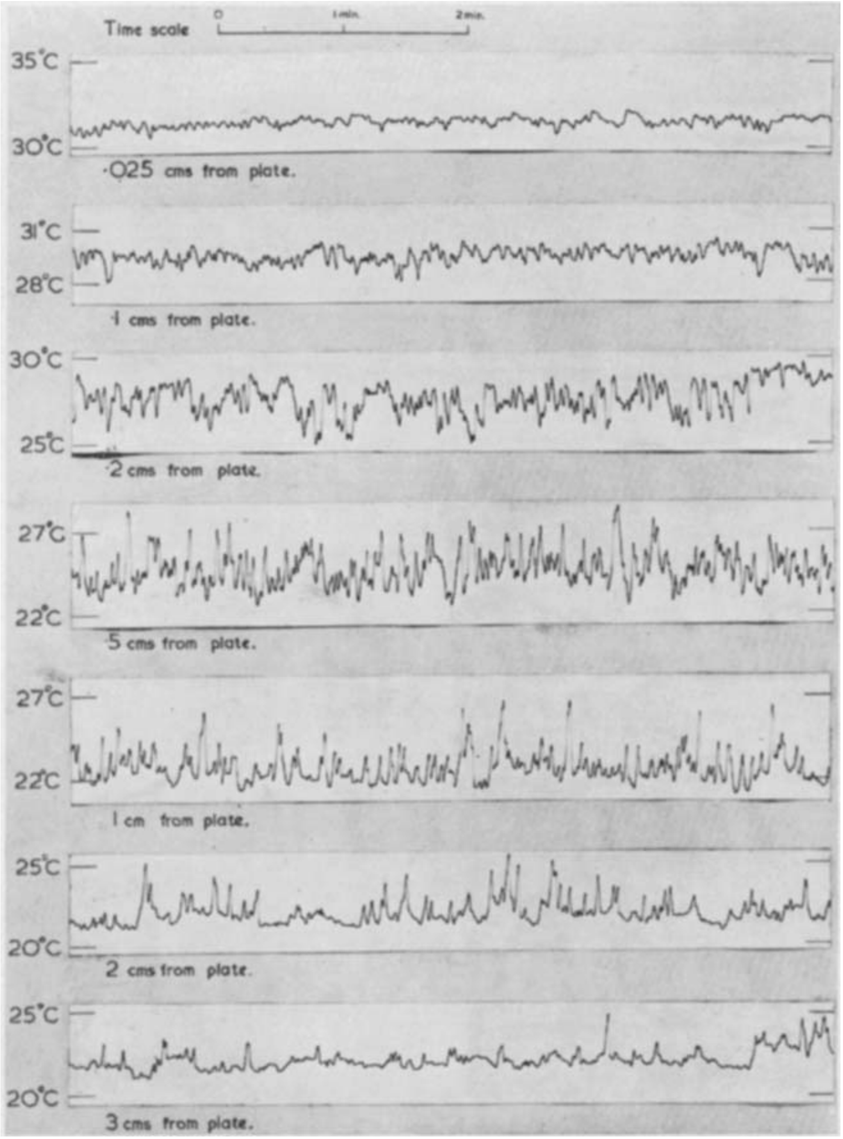

Figure 9. Temperature fluctuation records over a heated horizontal plate ($T_1 = 31{\cdot}6°$ C, $T_R = 21{\cdot}6°$ C).

fluxes. It will be noticed that the measured coefficients of heat transfer are rather less than those found by Malkus (1954 a) who used water and acetone, but that the difference seems to decrease with increasing Rayleigh number.

The measurements of mean temperature are shown in figures 3 to 5. It will be noticed that the profile at the highest Rayleigh number is markedly asymmetrical and even contains a section with reversed temperature gradient.

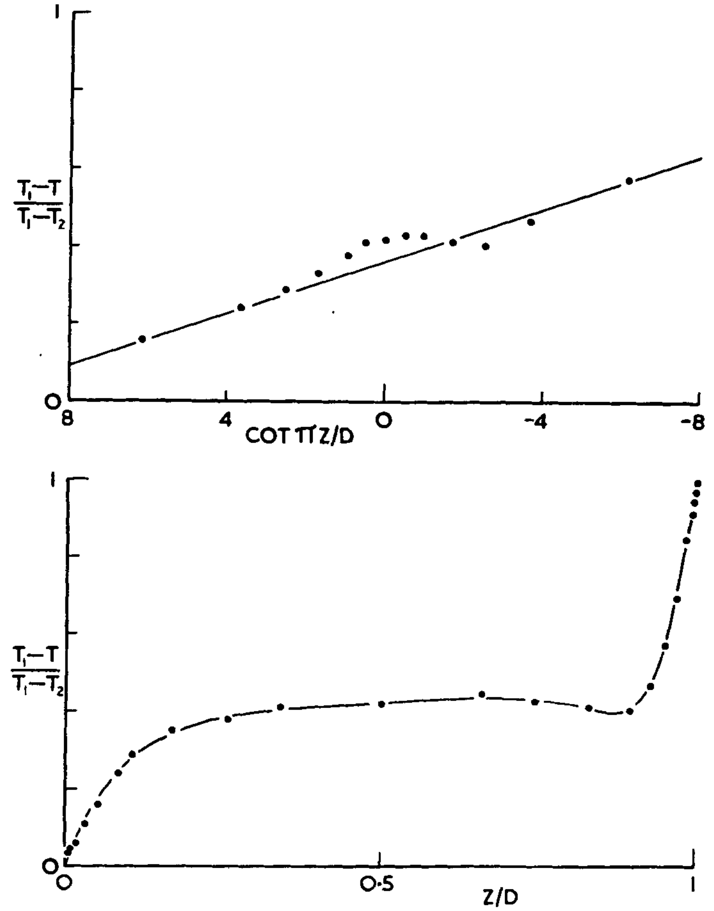

Figure 5. Distribution of mean temperature between parallel plates (D = 5·87 cm, T_1 = 46·5° C, T_2 = 9·7° C, A = 6·75 × 10^5).

It is believed that this is due to the onset of a circulatory motion of dimensions similar to those of the whole apparatus, with air rising from the centre of the lower plate where the measurements were taken. This motion would cause a finite mean shearing stress at the boundaries and logarithmic distributions of mean velocity and temperature may be expected. If the mean temperature is plotted against the logarithms of the distances from the upper and lower surfaces (figure 6), substantial regions of logarithmic variation are found.

Measured quantities				Derived quantities							Malkus (1954 a)
D	T_1	T_2	$H\times10^{-4}$	$(T_1 T_2)^{1/2}$	$\log T_1/T_2$	ν	κ	$A\times10^{-5}$	$Q\times10^{2}$	$\frac{QD}{\kappa \log (T_1/T_2)}$	$\frac{QD}{\kappa \log (T_1/T_2)}$
2·92	52·4° C = 325·6° K	12·3° C = 285·5° K	8·5, 11·2, 8·3	304·9° K	0·132	0·164	0·230	0·85	2·65	2·58	3·6
4·88	46·8° C = 320·0° K	10·6° C = 283·8° K	10·2, 9·3, 9·2	301·4° K	0·121	0·161	0·226	3·77	2·73	4·91	5·3
5·87	46·5° C = 319·7° K	9·7° C = 282·9° K	10·5, 14·0, 5·6	300·7° K	0·123	0·160	0·225	6·75	2·90	6·11	6·1

Table 1. Heat transfer measurements (c.g.s units). The values of ν and κ at temperature $(T_1 T_2)^{1/2}$ are taken from Montgomery (1947). The first column of heat transfers is obtained from the total heat loss by subtracting the computed losses by radiation and by conduction along the spacing pieces. The second column gives the two values of the transfer computed from the temperature gradients at the upper and lower surfaces respectively. The value used for the derived quantities is the mean of these three values.

If the other two profiles are plotted in like manner, there is no sign of a linear section on the curves. The onset of this circulation at a Rayleigh number around $5 \cdot 10^5$ is probably due to the side walls forcing the large eddies of the convection to remain relatively fixed in position. It is probable that,

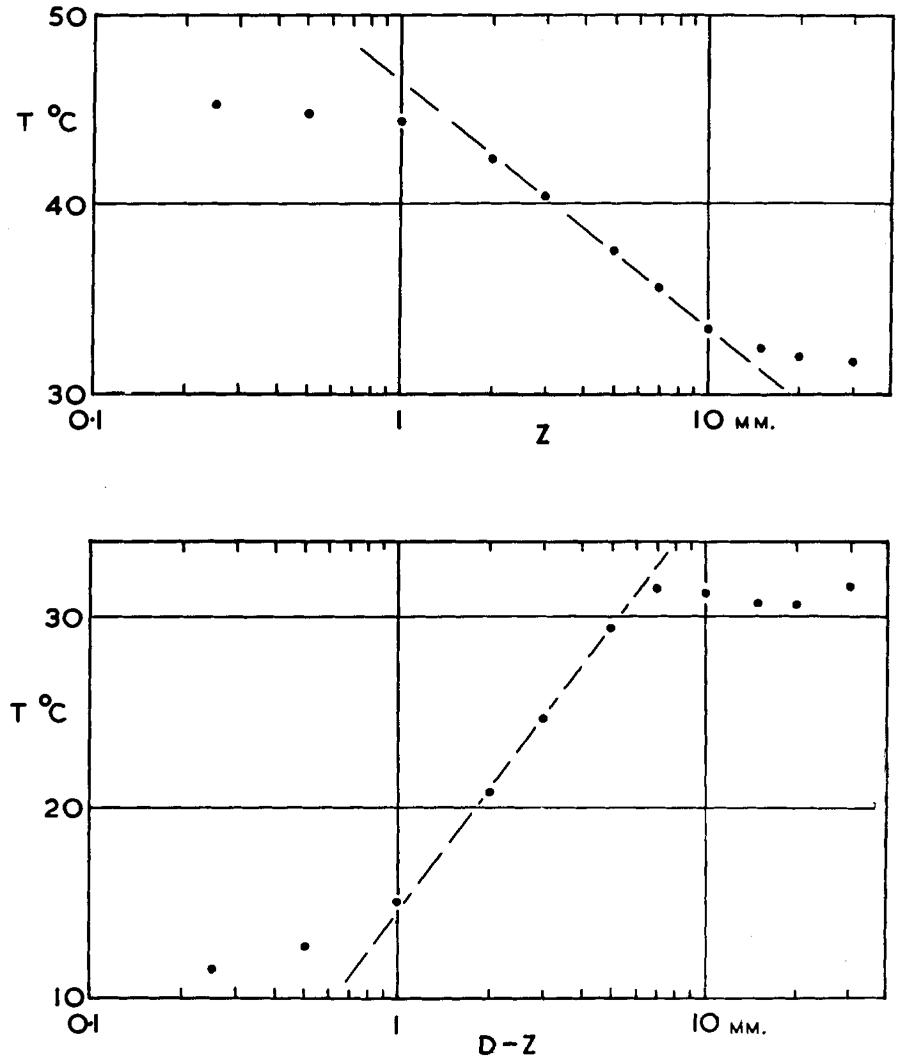

Figure 6. Logarithmic temperature profiles ($A = 6{\cdot}75 \times 10^6$).

for greater ratios of plate width to separation, these eddies could wander freely and then measurements at a fixed point would be identical with spatial means—which clearly they are not, for this particular profile.

For comparison with the Malkus theory, the three profiles have been plotted against $\cot(\pi z/D)$ and straight lines fitted to the central portion (figure 3 to 5). At the lower Rayleigh number there is a substantial linear region but the slope is less than the predicted value. The slope of the second profile agrees more closely with the prediction, but substantial departures from linearity exist. As shown by this method of plotting, these departures are similar in kind to the gross distortion in the third profile and may be caused by a weak circulation of large scale.

Rayleigh Number	$\frac{2\pi QD}{\kappa \log(T_1/T_2)}$	(Measured slope)$^{-1}$
$0{\cdot}85 \times 10^5$	16·1	12·4
3·77	30·9	35
6·75	38·5	30

Table 2. Comparison of equation (3·10) with experiment.

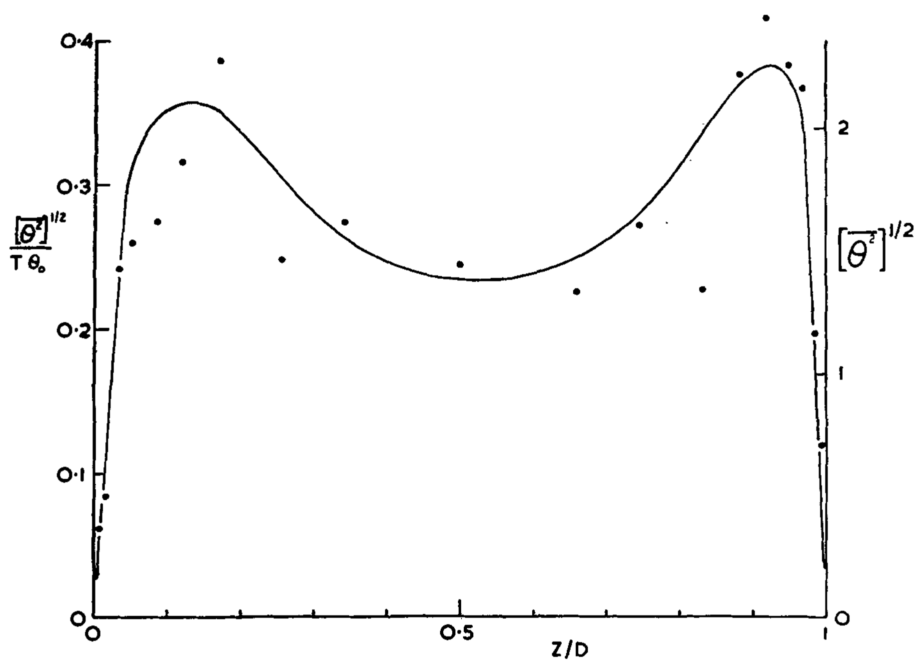

Figure 7. Temperature fluctuation intensities between parallel plates ($A = 6{\cdot}75 \times 10^5$).

Measurements of the root-mean-squares of the temperature fluctuations for the highest Rayleigh number are shown in figure 7. The average level is 1·9° C, compared with the value of 4·9° C given by equation (3.11). This discrepancy is probably too large to be explained by the finite time of response of the recording equipment. As would be expected from the shape of the mean temperature profile, the intensity of the temperature fluctuations reaches a peak just outside the conduction layer,

Figure 8 shows the autocorrelation function of the temperature fluctuations, defined as

$$R(\tau) = \overline{\theta(t)\theta(t+\tau)}/\overline{\theta^2} \tag{5.1}$$

at three values of z. It will be noticed that the time scale of the fluctuations decreases sharply as the lower surface is approached.

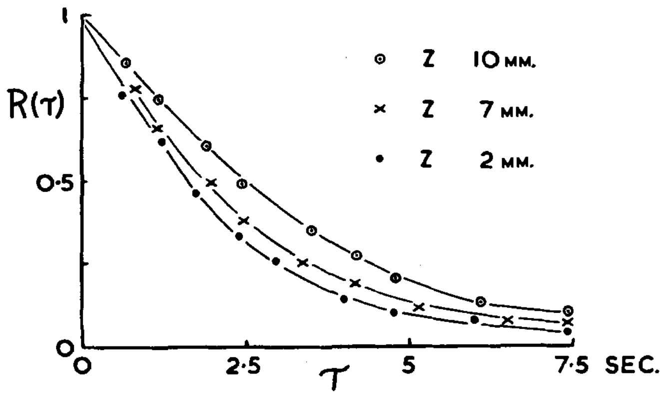

Figure 8. Autocorrelation functions of the temperature fluctuations (D = 5·87 cm, $T_1 - T_2$ = 38·8° C).

6. The convection over a single horizontal plane

Before discussing these measurements, it is necessary to consider the relation of the experimental arrangement—a horizontal heated plane forming the bottom of an open box of roughly cubical form—to the infinite heated plane considered in the theory. Unless there is appreciable large-scale circulation, and subsidiary experiments showed that there were not, the conditions near the surface will approximate to those near an infinite plane if the distance from the surface is small compared with distance from the walls and distance from the top of the box. With the single exception of the autocorrelation measurements, no measurements were made more than three centimetres from the surface and the condition is satisfied.

Measured quantities					Derived quantities			
T_1	T_R	$H \times 10^{-4}$	$H \times 10^{-4}$	T_a	$\log T_1/T_a$	$Q \times 10^2$	$\theta_0 \times 10^2$	z_0
31·6° C	21·6° C	0·535	0·503	21·6° C	0·0434	1·48	1·12	0·158
52·1° C	23·6° C	1·71	1·79	24·4° C	0·0917	4·99	2·80	0·116
72·1° C	22·1° C	3·38	3·09	28·5° C	0·1565	9·20	4·44	0·100

Table 3. Convection over a horizontal plane (c.g.s. units).

The boundary conditions of the theoretical flow are defined by the temperature of the plane and the temperature at infinity, the 'ambient temperature', which corresponds with the temperature at the base of the convection plume that rises from the top of the box and carries away the heat convected from the plane. Unfortunately, measurements of this base temperature, which may differ from room temperature by as much as 8° C, were not made at the time of the experiments, and it is necessary to estimate it by extrapolation of the measured temperature profiles. In figure 10, the three measured temperature profiles are plotted non-dimensionally as $\theta_0^{-1}\log(T/T_R)$ *vs* z/z_0, using the room temperature as reference temperature. It is clear that vertical displacement of the curves, corresponding to changes in the effective ambient temperature, will bring

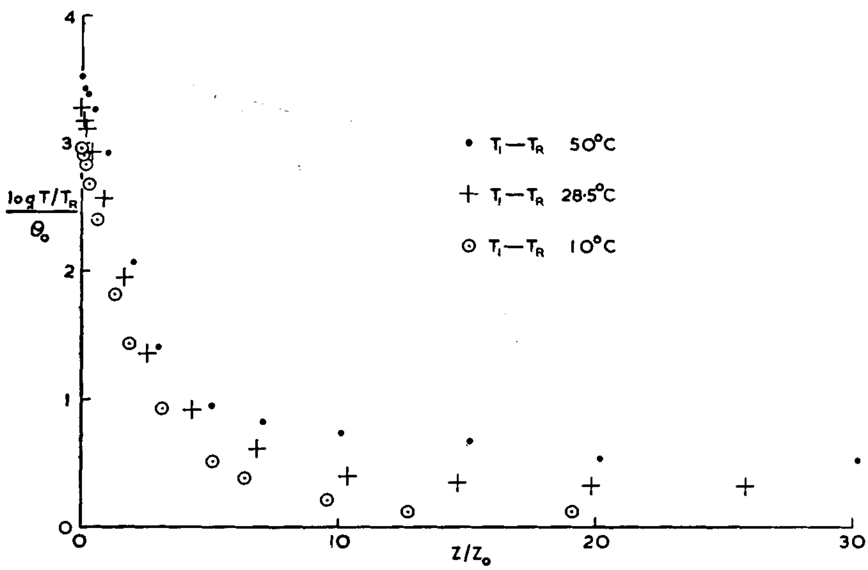

Figure 10. Distribution of mean temperature over a heated horizontal plate (similarity plot referred to room temperature).

them into near coincidence. The results plotted in figure 11 have been adjusted in this way, using the smallest possible positive differences of ambient temperature from room temperature, that is, zero for the first profile for which $T_1 - T_R = 10{\cdot}0°$ C. The necessary adjustment increases rapidly with plate temperature, probably because air currents in the room are more effective in ventilating the top of the box when the convection plume is weak.

The universal non-dimensional profile so obtained may be represented within experimental error by

$$\theta_0^{-1}\log(T/T_a) = 3{\cdot}0(z/z_0)^{-1} \qquad (6.1)$$

for all values of z/z_0 greater than two (see figures 11 and 12). It is not possible to represent the profile by a $z^{-1/3}$-law over a substantial range of z/z_0 without assuming ambient temperatures much less than room temperature and an upper limit to its validity at $z/z_0 = 12$. This upper limit would imply that the influence of the distant boundaries becomes dominant at only 1 cm from the plate, which seems highly unlikely, and that the effective ambient temperature is less than room temperature, which is equally unlikely.

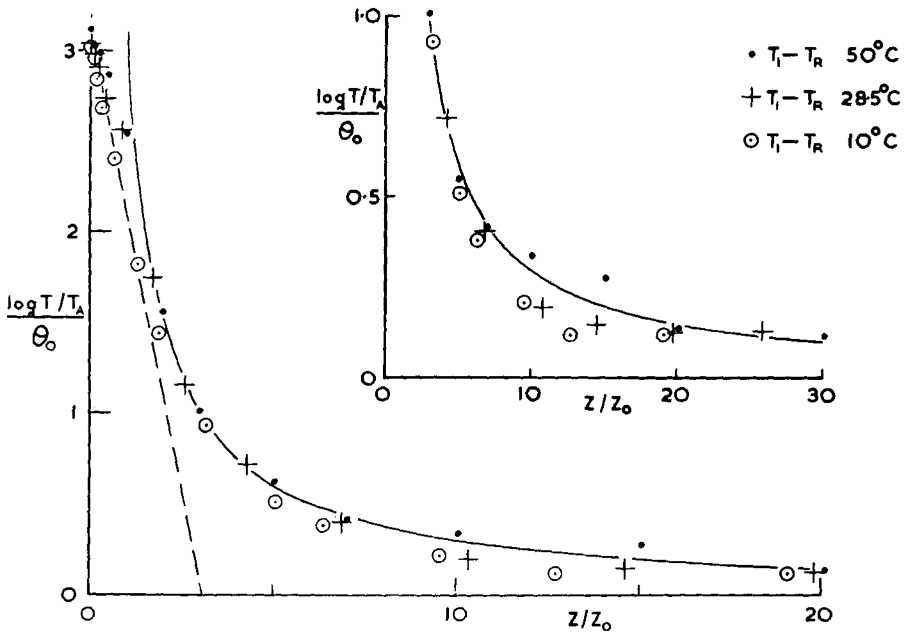

Figure 11. Distribution of mean temperature over a heated horizontal plate (similarity plot referred to a selected ambient temperature). The full line follows equation (6.1).

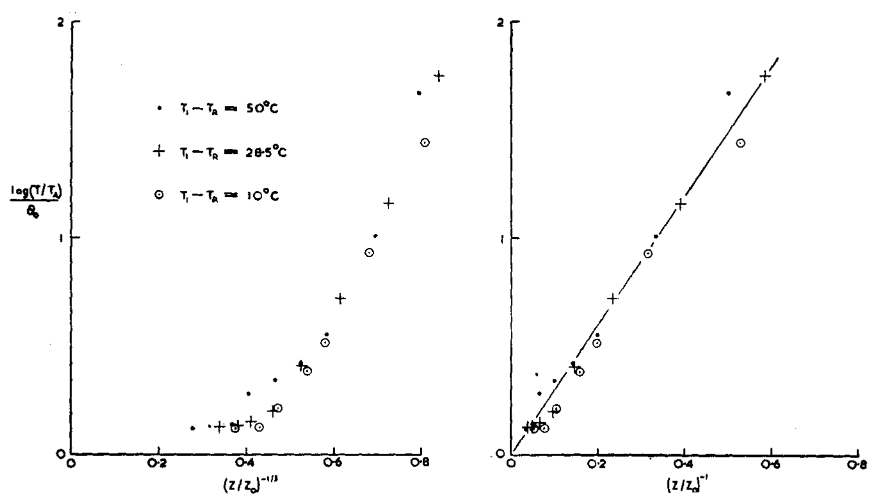

Figure 12. Mean temperatures plotted against $(z/z)^{-1}$ and $(z/z)^{-1/3}$ for $z/z_0 > 2$.

Direct measurements of heat transfer are shown by plotting first $\theta_0^{-1}\log(T_1/T_R)$ and $\theta_0^{-1}\log(T_1/T_a)$ against $\log(T_1/T_R)$ (figure 13). In the second graph, appropriate values of $T_a - T_R$ have been obtained by interpolating between the values used in figure 11. If the convection is really independent of the side walls and the distant boundaries, $\theta_0^{-1}\log(T_1/T_a)$ should be independent of temperature difference, as is implied by equation (3.3). With these values for the ambient temperature, these measurements indicate that

$$\log\frac{T_1}{T_a} = 3{\cdot}4\theta_0 = 3{\cdot}4\left(\frac{Q^3}{\kappa g}\right)^{1/4}, \tag{6.2}$$

nearly independent of temperature difference. The coefficient 3·4 may be compared with the value 3·1 obtained from the temperature profiles.

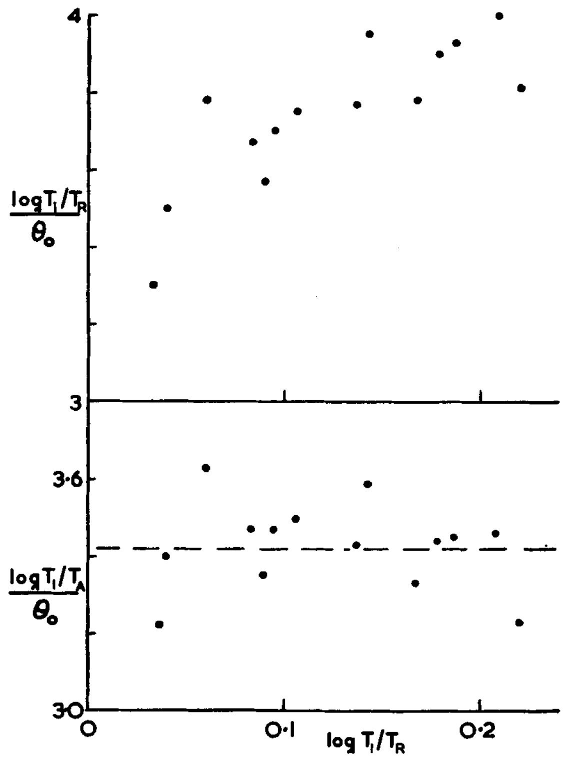

Figure 13. Measurements of heat transfer from a heated horizontal plate (plotted non-dimensionally).

It is also interesting to compare these values of heat transfer with measurements between parallel planes at large Rayleigh numbers. The measurements of Malkus (1954 a) and others are closely represented by

$$\frac{HD}{k(T_1 - T_2)} = \left(\frac{\lambda}{2000}\right)^{1/3}. \tag{6.3}$$

If the heat transfer at each surface were little affected by the presence of the other surface, we would expect the ratio

$$\frac{\log (T_1/T_a)}{(Q^3/\kappa g)^{1/4}}$$

to be a constant and the temperature midway between the surfaces to be the geometric mean of the surface temperatures. From equation (6.3) and inserting the value of the Prandtl number ν/κ for air, we find

$$\log \frac{T_a}{T_1} = 3{\cdot}12 \left(\frac{Q^3}{\kappa g}\right)^{1/4} \qquad (6.4)$$

($T_a = (T_1 T_2)^{1/2}$), in good agreement with the previous values*. Even the measurements of heat transfer of the previous section give values of 4·0, 3·7 and 3·6 at Rayleigh numbers of $0{\cdot}85 \times 10^5$, $3{\cdot}77 \times 10^5$ and $6{\cdot}75 \times 10^5$, although these are too low for independence of the two surface layers.

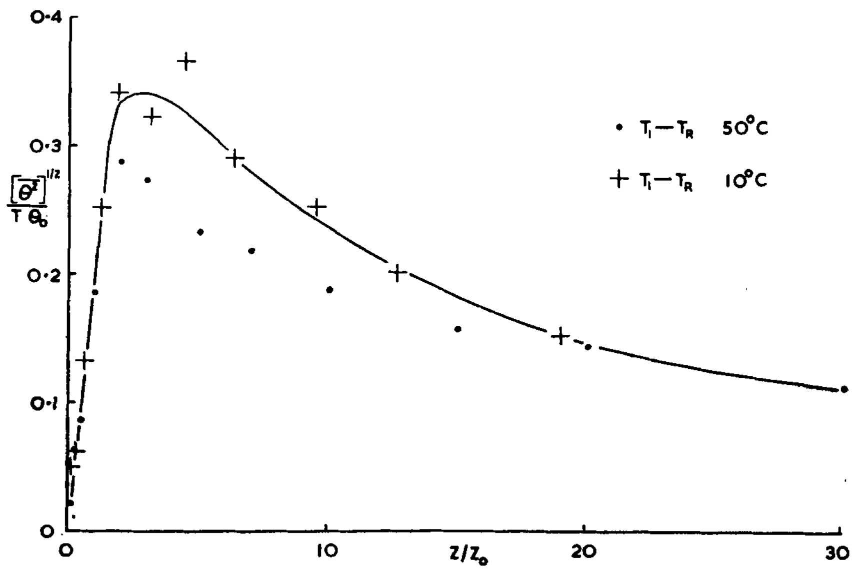

Figure 14. Temperature fluctuation intensities over a heated horizontal plate.

Measurements of root-mean-square temperature fluctuation, plotted non-dimensionally as $[\overline{\theta^2}]^{1/2}/[T(Q^3/\kappa g)^{1/4}]$ against $(gQ/\kappa^3)^{1/4}z$, are shown in figure 13. The measurements for the two heat transfers only agree at the larger values of $(gQ/\kappa^3)^{1/4}z$. It is very probable that the response time of the galvanometer prevented the complete recording of the rapid fluctuations occurring near the surface. The single measurement of the autocorrelation function (figure 14) shows that a substantial part of the intensity resides in

* These non-dimensional equations refer to air with $\nu/\kappa = 0{\cdot}77$. Accepting equation (6.3) as correct, the general form of equation (6.4) is

$$\log \frac{T_1}{T_a} = 3{\cdot}34 \left(\frac{Q^3}{\kappa g}\right)^{1/4} \left(\frac{\nu}{\kappa}\right)^{1/4}.$$

Fourier components of periods less than the period of the galvanometer, one second. It may be mentioned that more recent work using equipment with faster response has registered temperature fluctuations of much greater amplitude.

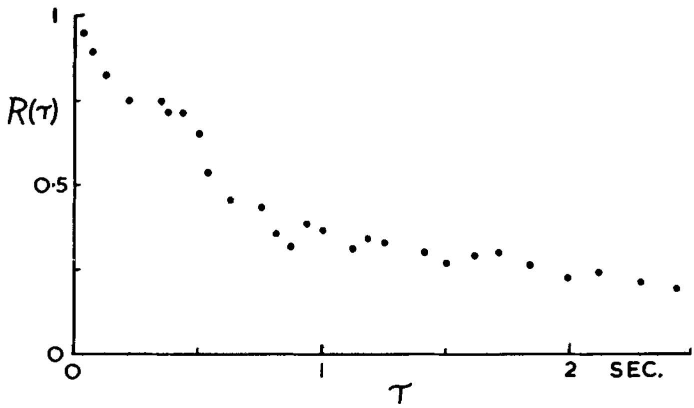

Figure 15. Autocorrelation function of the temperature fluctuations over a horizontal plate ($T_1 - T_R = 70{\cdot}9^\circ$ C, $z = 4$ cm).

7. Discussion of results

The most interesting feature of these measurements is the close correlation between heat transfer from a single surface and heat transfer between parallel planes. Even though the Rayleigh numbers of the parallel plane experiments were comparatively small, they tend to confirm the experimental finding of Malkus that, at high Rayleigh numbers, the heat transfer coefficient is nearly proportional to the cube root of the Rayleigh number and so is independent of the separation of the surfaces. This must mean that the conditions near each surface dictate the heat transfer and that the central region is a region of negligible temperature gradient, easily capable of transmitting the imposed heat flux. Nearly all the temperature drop occurs in the wall layers which have a temperature distribution determined by the heat transfer, the conductivity (and viscosity) and the gravitational field. Additional confirmation of this view is found in the observation of Malkus (1954 a) that the heat transfer between parallel planes is practically unaffected by the presence of vertical partitions even though they restrict greatly the motion of the central region.

Although both the existing theories of convection conform to this general pattern, neither is capable of describing accurately the measured profiles. The similarity theory (Priestley 1954) requires that $\log(T/T_a)$ should be proportional to $z^{-1/3}$ for *all* large values of z/z_0, where T_a, the temperature at large distances from the surface, is certainly not less than room temperature. It is impossible to fit any substantial part of the observations by such a power law without using a value of T_a less than room temperature.

The very interesting theory of Malkus (1954 b) is more successful in as much as its prediction that

$$\log \frac{T}{T_a} = C\theta_0 \frac{z_0}{z} \tag{7.1}$$

fits the observations well if $C = 3{\cdot}0$. However, consistency of the heat transfer measurements and the theory requires that $C = 2{\cdot}0$ (see equation (3.9)).

The failure of the similarity theory to describe the temperature variation within the range of measurement implies that the local temperature gradients and other local characteristics of the convection are affected either by the conductivity (or viscosity) of the fluid or by the convection near the distant boundary. This may appear surprising in view of the considerable success of the similarity treatment of flow in the constant stress region of turbulent wall flow, for which similar assumptions of independence of viscosity and presence of distant boundaries are made. The analogy fails in that the constant stress layer is a layer of substantially uniform turbulent intensity in which no appreciable net transport of turbulent energy is expected, while the surface layer, as described by the similarity theory, would have a turbulent intensity varying as $(Qgz)^{2/3}$ and substantial transport of turbulent energy towards the surface would be expected. Also, the intensity of the temperature fluctuations varies considerably and they will be transported through the surface layer. These considerations (and the present results) suggest that interaction between different parts of the surface layer may be more effective than the similarity theory supposes. It is possible that the conditions assumed may exist for values of z/z_0 beyond the range of measurement, where the difference from ambient temperature is very small compared with the temperature difference across the whole layer. If the meteorological evidence is accepted, it presumably means that these measurements were made in such a region.

The defects in the Malkus theory are difficult to diagnose, partly because the theory is based on general statements not easily related to the more common approach through the equations of motion and heat. The difference between the theoretical and observed distributions of temperature in the surface layer indicates that the convection has not attained the optimum condition specified in the theory. In fact, it seems to have settled down with a heat transport about 70% of the maximum set by the theory. Two possible causes are (*a*) that the requirement that the minimum temperature gradient is zero is insufficiently restrictive, and (*b*) that the Fourier series used to describe the convection does not satisfy all the boundary conditions. In the central region at the higher Rayleigh numbers, there is a systematic tendency to lower gradients, which is a departure in the opposite direction. Some of this may be due to the presence of a large-scale circulation, but measurements of the closely analogous flow between concentric rotating cylinders show an extensive region of constant angular momentum, the quantity analogous to temperature in convection (Taylor 1935). Pai (1943) finds that the gradient of angular momentum is reversed in two small regions adjacent to each cylinder, and explains this as a result of large-scale circulations which are part of the general flow. It may well be that the large-scale circulation postulated to explain the form of the temperature profile at the largest Rayleigh number is typical of the flow as a whole, except in its position, which is determined by the presence of side walls. For the present, it may be concluded that the Malkus theory is in fair but not good agreement with experiment, which is a remarkable achievement for a theory of turbulence not containing any disposable constants.

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